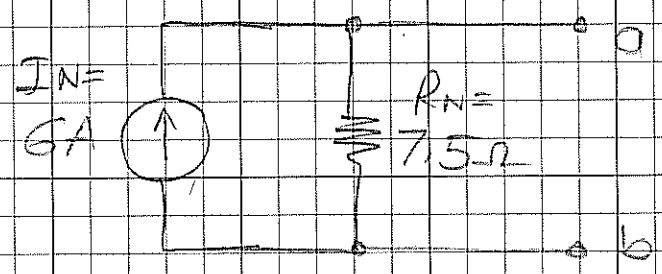
So the open circuit voltage is:

$$V_{oc} = 12 \times 3.75 = 45\,V$$

$$R_N = \frac{V_{oc}}{I_{sc}} = \frac{45}{6} = 7.5\,\Omega$$

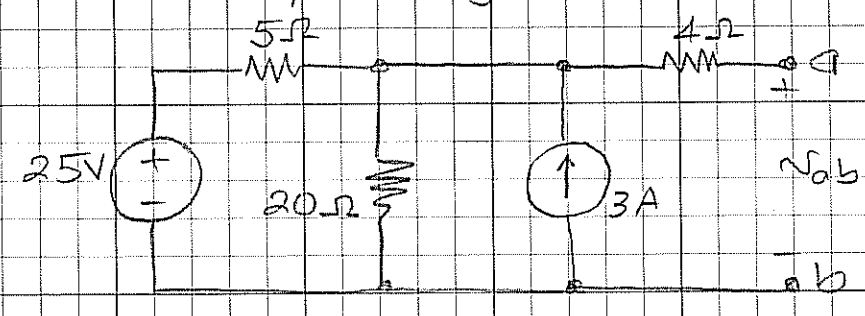
So the Norton Equivalent is:



4.11 More on Deriving a Thévenin Equivalent:

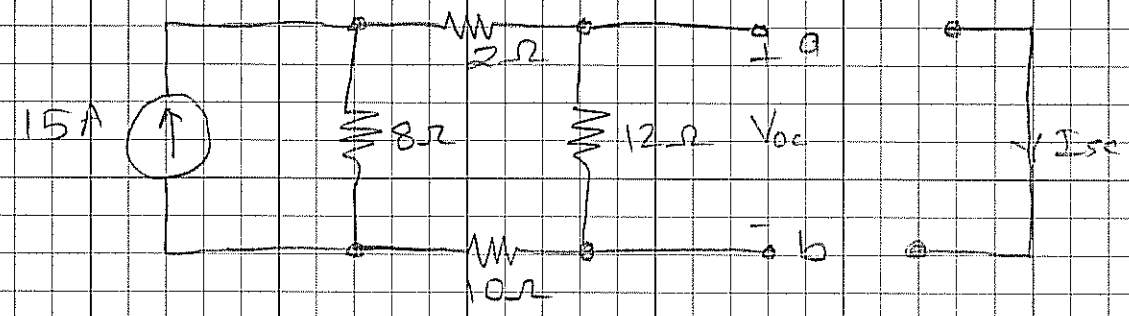
When the network contains only independent sources, we can deactivate the sources and then calculate the resistance seen between the designated pair. A voltage source is deactivated by replacing it with a short circuit and a current source is deactivated by replacing it with an open circuit:

Consider the following circuit



Assessment Problem 4.17

Find the Norton Equivalent with respect to terminals "a" and "b" in the following circuit:



$I_N = I_{sc}$ and $R_N = \frac{V_{oc}}{I_{sc}}$

Under SC the 12Ω resistor is shorted! The short circuit current is the current in 2Ω (and the 10Ω) resistor. This current can be found by the current division rule:

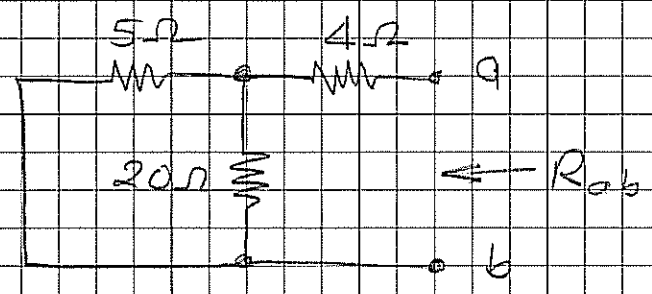
$$I_{sc} = 15 \times \frac{8}{8+(2+10)} = 6\,A \quad \left(= 15 \times \frac{48}{12}\right)$$

$I \times R_{eq}/R_i$

The open circuit voltage is the voltage across the 12Ω resistor, which can be deduced from the current that flows in 12Ω resistor. This current can also be found using current division:

$$I_{12\Omega} = 15 \times \frac{8}{8+(2+12+10)} = 3.75\,A$$

When the sources are deactivated, we obtain the following circuit:

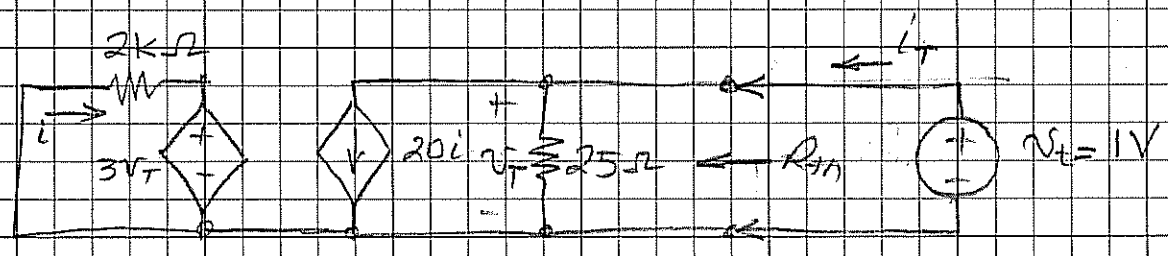


Then $R_{ab} = 4 + \frac{20 \times 5}{25} = 8\Omega$

Finding the Thevenin using an external Source:

When the circuit contains dependent sources, and no independent sources, its Thevenin impedance may be found using an external test source:

Find the Thevenin equivalent resistance of the following circuit:



Connect an external test $v_T = 1V$ and calculate i_t. The ratio v_T / i_T is the Thevenin resistance.

$$i = -\frac{3v_T}{2} = -\frac{3}{2} = -1.5\,mA$$

The current source has a current:

$$20i = 20 \times (-1.5) = -30\,mA$$

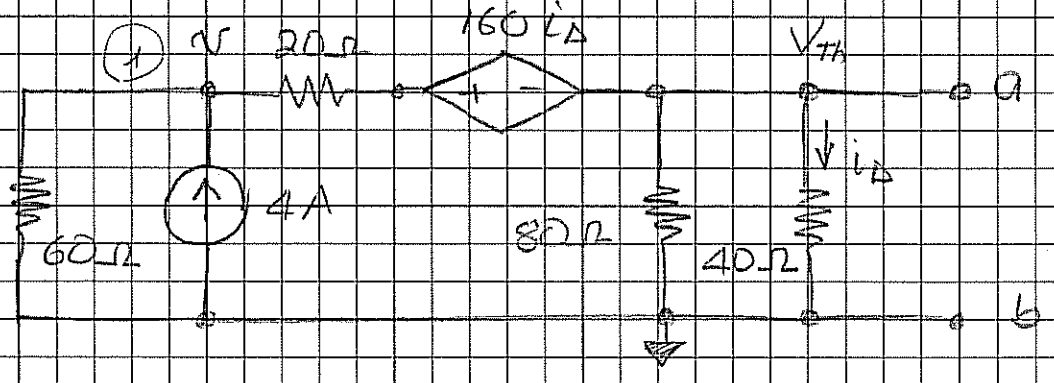
The current i_T is given by:

$$i_T = \frac{1}{25} - 0.03 = 0.01\ A$$

So $R_{Th} = \frac{v_T}{i_T} = \frac{1}{0.01} = 100\Omega$

Assessment Problem: 4.20

Find the Thevenin equivalent seen at terminals a and b:



Solve using the node voltage method:

$$\frac{v}{60} - 4 + \frac{v - (160 i_\Delta + V_{Th})}{20} = 0 \quad (1)$$

$$\frac{V_{Th}}{40} + \frac{V_{Th}}{80} + \frac{(V_{Th} + 160 i_\Delta - v)}{20} = 0 \quad (2)$$

Rearrange (1) and (2) replacing i_Δ by $\frac{V_{Th}}{40}$:

$v = 172.475\ V \qquad V_{Th} = 30V$

To find the resistance, we need to calculate the short-circuit current.

KVL at node 1 with a and b shorted. Note that $i_\Delta = 0$:

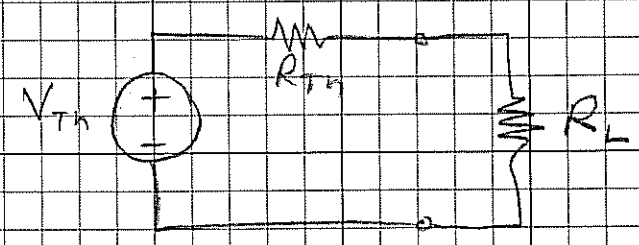
$$\frac{v}{60} - 4 + \frac{v}{20} = 0 \Rightarrow v = 60\,V$$

$$I_{sc} = \frac{v}{20} = \frac{60}{20} = 3A$$

The Thevenin resistance = $R_{Th} = \frac{30}{3} = 10\Omega$

4.12 Maximum Power Transfer

The maximum power that can be delivered by circuit can be determined using its Thevenin equivalent:



For the above network, what is the value of R that causes the power delivered to R_L to be maximum. The power delivered to R_L is:

$$P = i^2 R_L = \left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 \times R_L$$

P is maximized when its derivative is zero:

$$\frac{dP}{dR_L} = V_{Th}^2 \frac{(R_{Th}+R_L)^2 - 2R_L(R_{Th}+R_L)}{(R_{Th}+R_L)^4} = 0$$

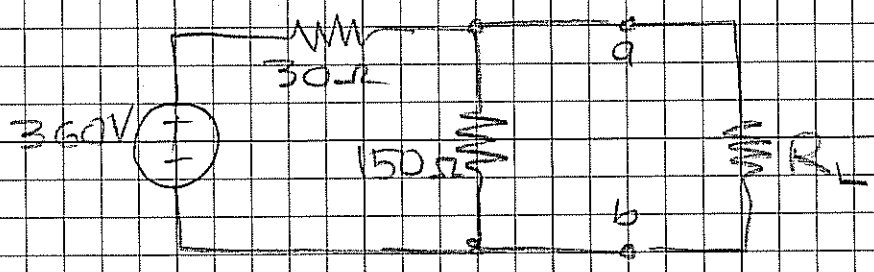
$$R_{Th}+R_L-2R_L = 0 \Rightarrow R_L = R_{Th}$$

Replace in the expression of P:

$$P_{max} = \left(\frac{V_{Th}}{2R_{Th}}\right)^2 R_{Th} = \frac{V_{Th}^2}{4R_{Th}}$$

Example 4.12

Find the value of R_L that results in maximum power transfer to R_L. Find P_{max}.



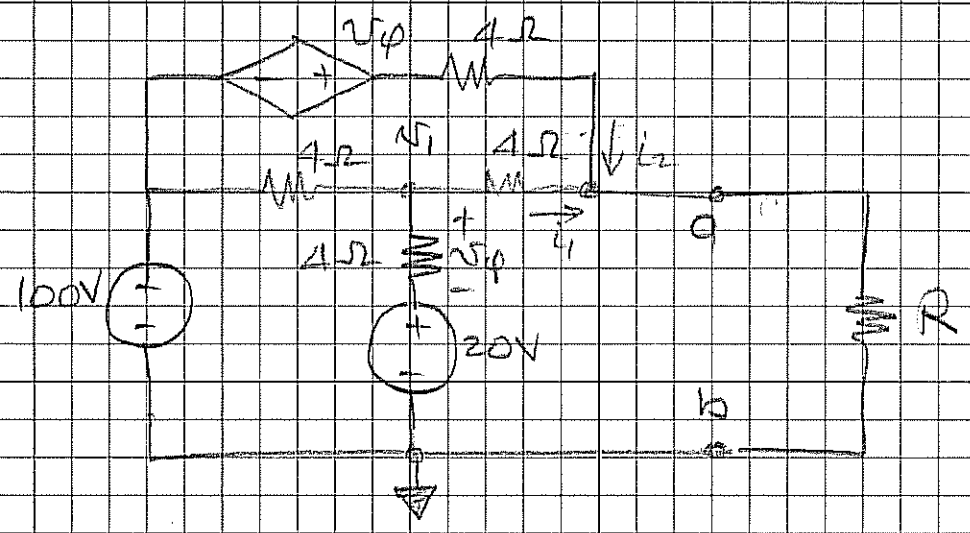
The load resistance R_L should be equal to the Thevenin resistance between a and b:

$$R_{Th} = \frac{30\times150}{180} = 25\,\Omega \qquad V_{Th} = 360\times\frac{150}{180} = 300$$

$$P_{max} = \frac{300^2}{4\times25} = 900\ W$$

Assessment Problem 4.21

Find the value of R for maximum power to be delivered to terminals a and b. Find P_{max}.



We need to determine the Thévenin equivalent between terminals a and b.

Remove R (open circuit) and calculate V_{oc}:
Under O.C. There are two essential nodes. Select the bottom one as reference and write KCL for node 1 to determine v_1:

$$\frac{v_1-100}{4}+\frac{v_1-20}{4}+\frac{v_1-(100+v_\varphi)}{8}=0$$

v_1-20

but $v_\varphi = v_1 - 20$ so:

$$\left(\frac{1}{4}+\frac{1}{4}+\frac{1}{8}-\frac{1}{8}\right)v_1 = 25+5+10 \Rightarrow v_1 = 80\text{V}$$

The open circuit voltage is calculated as v_1 minus the voltage drop across the 4Ω resistor:

$V_{oc} = v_1 - 4i_1$ but $i_1 = \dfrac{v_1 - (100 + v_\phi)}{8}$

$v_\phi = v_1 - 20$

So $i_1 = -10A$

$V_{oc} = 80 - 4(-10) = 120V = V_{Th}$

Replace R by short circuit and solve for v_1:

$$\frac{v_1 - 100}{4} + \frac{v_1 - 20}{4} + \frac{v_1}{4} = 0 \Rightarrow v_1 = 40V$$

The current in the short circuit is $i_1 + i_2$:

$$i_1 = \frac{v_1}{4} = \frac{40}{4} = 10A$$

$$i_2 = \frac{100 + v_\phi}{4} = \frac{100 + 40 - 20}{4} = 30A$$

So $I_{sc} = i_1 + i_2 = 40A$

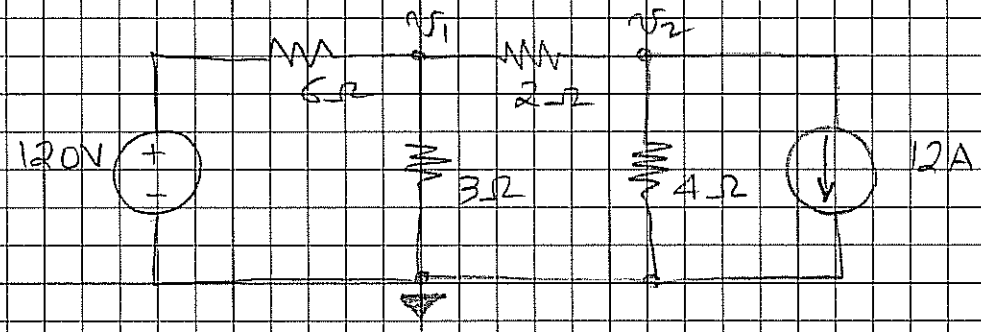
$$R_{Th} = \frac{V_{oc}}{I_{sc}} = \frac{120V}{40A} = 3\Omega$$

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{120^2}{4 \times 3} = 1200W = 1.2kW.$$

4.13 Superposition

The principle of superposition states that when a linear system is excited by more than one independent source, the total response is the sum of the individual responses.

Superposition is demonstrated to find the branch current in the circuit below, which has two sources:



We will calculate the branch currents when the 12A C.S. is deactivated, and then the branch currents when the 120V source is deactivated. The net current in each branch is the algebraic sum of the two currents obtained. Similarly the node voltage at any node is the sum of node voltages obtained in the two solutions:

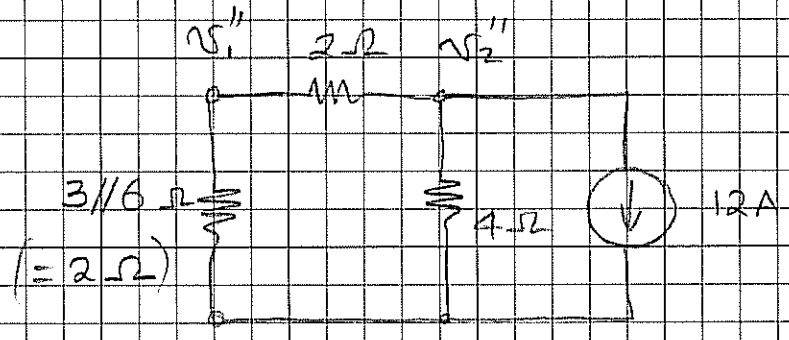
i) Deactivate the 12A source (replace it with an open circuit). The circuit is solved by one node-voltage equation:

$$\frac{v_1' - 120}{6} + \frac{v_1'}{3} + \frac{v_1'}{(2+4)} = 0 \Rightarrow v_1' = 30V.$$

The voltage v_2' is calculated using the voltage-division rule:

$$v_2' = v_1' \times \frac{4}{6} = 20\,V$$

ii) Deactivate the 120V source (with the 12A active again). Here v_2'' will be determined using the NVM:



$$\frac{v_2''}{4} + \frac{v_2''}{4} + 12 = 0 \Rightarrow v_2'' = -24\,V$$

$$v_1'' = v_2'' \times \frac{2}{4} = -12\,V$$

Using "superposition"

$$v_1 = v_1' + v_1'' = 30 - 12 = 18\,V$$

and $v_2 = v_2' + v_2'' = 20 - 24 = -4\,V$

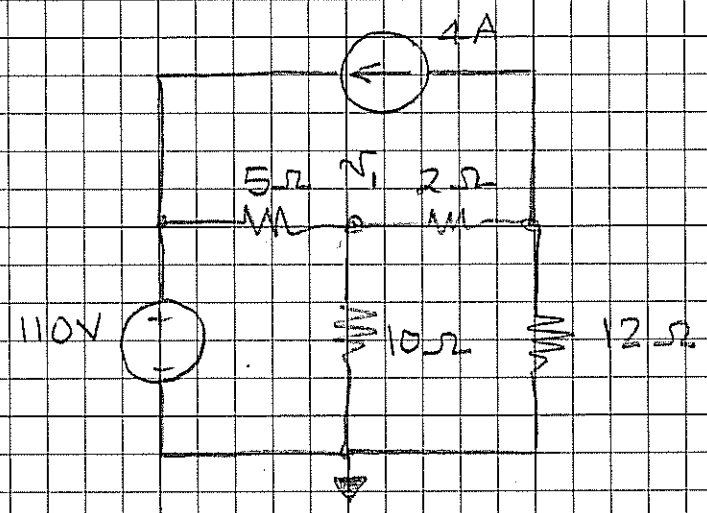
$$i_{3\Omega} = i_3' + i_3'' = \frac{v_1'}{3} + \frac{v_1''}{3} = \frac{v_1}{3} = 6\,A$$

$$i_{4\Omega} = i_4' + i_4'' = \frac{v_2'}{4} + \frac{v_2''}{4} = \frac{v_2}{4} = -\frac{4}{4} = -1\,A$$

Example 4.13: Superposition with dependent sources (self study). Note that dependent sources cannot be deactivated!

Problem 4.91

Find v in the following circuit using superposition and the power dissipated in the 10 Ω resistor:

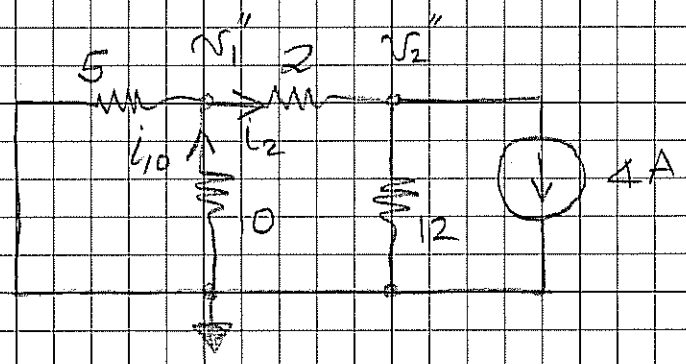


First deactivate the 4A source (open circuit) and find v_1' using voltage division:

$$v_1' = 110 \times \frac{10 \,//\, (2+12)}{5 + 10 \,//\, (2+12)}$$

$$= 110 \times \frac{5.8333}{5 + 5.8333} = 59.23\,\text{V}$$

Now deactivate the 110V and solve for v_1. When the voltage source is deactivated, the circuit becomes:



By current division the current in the 2Ω resistor is:

$$i_2 = 4 \times \frac{12}{12 + \underbrace{(2 + 5 \| 10)}_{5.333}}$$

$$= 2.7692 \text{ A}$$

Also i_{10} can be found the current division:

$$i_{10} = i_2 \times \frac{5}{15} = 0.9230787 \text{ A}.$$

$$v_1'' = -i_{10} \times 10 = -9.230787 \text{ A}.$$

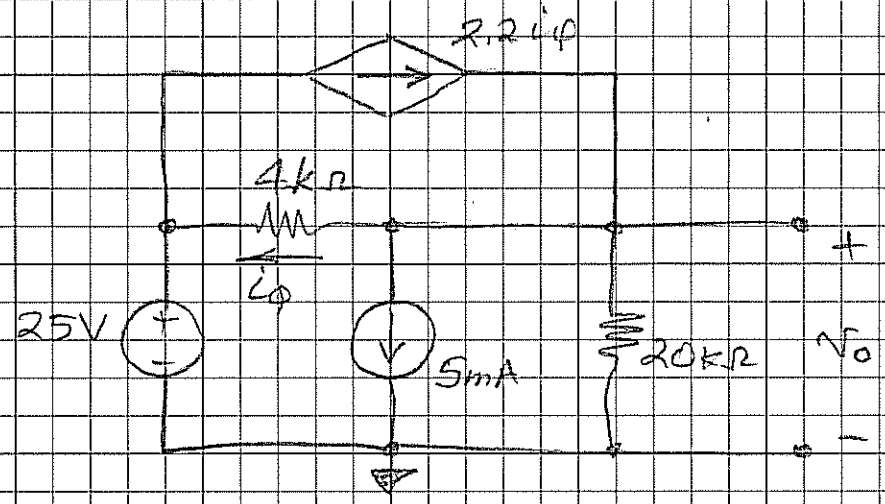
The voltage v_1 by superposition:

$$v_1 = v_1' + v_1'' = 59.23 - 9.23 = 50 \text{ V}$$

Power dissipated in the 10Ω resistor:

$$P_{10\Omega} = \frac{50^2}{10} = 250 \text{ W}.$$

Problem 4.96: Use superposition to determine v_o in the following circuit:



i) Replace the 25V by a S.C. (deactivate it), then write NVM for v_o':

$$\frac{v_o'}{20} + 5 + \frac{v_o' - 25}{4} - 2.2\, i_\varphi = 0$$

but $i_\varphi = \frac{v_o' - 25}{4}$, so replace in above equation and solve for v_o:

$$\frac{v_o'}{20} + 5 - 1.2\left(\frac{v_o' - 25}{4}\right) = 0$$

$$-0.25 v_o' - 2.5 = 0 \Rightarrow v_o' = 10\text{ V}$$

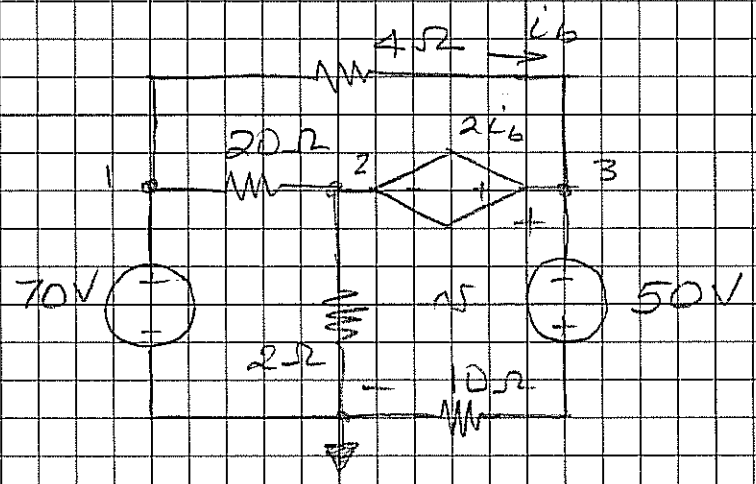
ii) Replace the 5mA source by an O.C. (Deactivate it). Again using NVM:

$$\frac{v_o''}{20} + \frac{v_o'' - 25}{4} - 2.2\left(\frac{v_o'' - 25}{4}\right) = 0 \Rightarrow$$

$$-0.25 v_o'' - 7.5 \Rightarrow v_o'' = -30\text{ V}$$

So $v_o = v_o' + v_o'' = 10 - 30 = -20\text{ V}$

Problem 4.97: Use the principle of superposition to find the voltage v.



The problem is easier to solve by a direct method. The MCM involves 3 equations in 3 unknowns. The NVM involves 2 equations in 2 unknowns v_2 and v_3 ($\triangleq v$). One equation will be that of the supernode 2-3 and the second one will be the constraint equation of the dependent voltage source.

NVM of supernode 2-3:

$$\frac{v_2-70}{20}+\frac{v_2}{2}+\frac{v_3+50}{10}+\frac{v_3-70}{4}=0 \quad (1)$$

The constraint equation of dependent voltage source i_b:

$$2i_b=2\left(\frac{70-v_3}{4}\right)\triangleq v_3-v_2 \quad (2)$$

Arrange equation (1) and (2) into standard form:

$$0.55v_2 + 0.35v_3 = 16 \quad (1)$$

$$-v_2 + 1.5v_3 = 35 \quad (2)$$

$$v_2 = 10\,V \quad v_3 = 30\,V \; (= v)$$

Problem 4.98:

Hint: Two Y-Δ transformations on the 1-2-8 Ω resistors are done on the top and bottom sides. This will be followed by two other Y-Δ transformations leaving the nodes of the voltage source and the nodes of the 4 Ω resistor apparent in the circuit.

A bridge finally appears and the bridge is balanced!

So the current $i = 0$!